THE DEVELOPMENT OF ARITHMETIC SKILLS IN CHILDREN PART I

ABSTRACT

The arithmetic skills are one of the most important abilities that must be mastered by a child, the numbers being frequently used in the daily living. Contrary to the piagetian theory, recent studies have demonstrated that infants have the capacity to perceive the numerosity of a set of objects and they are even able to "resolve" simple additions and subtractions. The next arithmetic acquisitions are based on the counting ability and the development of abstract thinking contribute to the understanding of arithmetical concepts and principles. From an anatomical point of view, as the children grow, the inferior parietal cortex specializes in mental arithmetic, and the calculation depend less on the working memory and attention processes that are sustained by the prefrontal cortex. The second part which we will publish it in the next number of the journal, will continue with pathological aspects of arithmetical abilities.

Key words: arithmetic skills, mathematical cognition.

Although the arithmetic skills are one of the most important cognitive functions that must be mastered by a child, there is not much data on the neural basis of the development of mathematical abilities.

Piaget developed one of the first theories regarding the arithmetic processing in children, proposing that the newborn babies have no numerical capacities (Piaget, 1952). In his view, the abstract arithmetic knowledge necessitates the development of logic, being developed around 4 – 7 years. In concordance with the piagetian theory, Isaacs sustained that arithmetical ability and the intellectual growth in general must be learned by the internalization of actions (Isaacs, 1960). In other words, before we can think on numerals we need to have acted with them many times in our daily living.

Contrary to these theories, the later studies demonstrated that infants have the capacity to perceive the **numerosity (the number of objects in a group independently of their physical proprieties).** They can correctly represent the exact number of objects in a scene, the limit being 3 or 4 objects (Feigenson, 2002). Furthermore, they are able to represent and compare the approximate values of a large number of objects, capacity that increases with age (Lipton & Spelke, 2003).

Starkey and Cooper used the habituation – dishabituation paradigm, an non-associative learning technique implying the progressive decrease of the behavioral response after repetitive exposure to a stimuli; they demonstrated that 4 – 6 month-old infants were sensitive to the numerosity change of an array of dots (Starkey & Cooper, 1980). In their experiment, repeating the same pattern of dots determined the babies to habituate and lose interest on the stimuli, but presenting a new pattern of dots caused them to regain interest (to dishabituate).

After this experiment, other researchers, in order to assure that the babies responded to numerosity and not to other features of the stimuli, used a variety visual stimulus, with different shapes and spatial arrangements, which were static or dynamic (Antel & Keating, 1983; Starkey et al., 1990; Van Looosbroek & Smitsman, 1990). Furthermore, it was demonstrated that newborn babies have the ability to discriminate not only the numerosity of visual stimuli, but also auditory stimuli (Bijeljac – Babic et al., 1993).

These studies were crucial, demonstrating that in the first year of life babies already have abstract numeric concepts, regardless of the presentation modality of the stimulus. Furthermore, it was demonstrated that the ability to discriminate stimuli is ratio-dependent in infants, similarly with adults. Thereby, 6 months-old babies necessitate a 1 : 2 ration, the 9 months-old – a 2 : 3 ratio, their precision improving with age (Xu & Spelke, 2000; Lipton & Spelke, 2003; Lipton & Spelke, 2004).

Another important issue is the infant's ability to "resolve" arithmetical problems. The first study regarding their capacity to add or to subtract was done by Wynn in 1992 (Wynn, 1992). Infants of 4 – 5 months-old were shown a doll that was placed on a scene; the doll was covered by a screen and the babies watched the examiner placing a second doll behind the screen. When the screen was removed, the infants looked longer when there were 1 or 3 dolls then when there were 2 dolls, demonstrating that their expectations were violated ("violation of expectation paradigm"). This experiment demonstrated for the first time the infant's ability to calculate; subsequent studies replicated these results (Simon et al., 1995; Koechlin et al., 1999; McCrink & Wynn, 2004).

The development of the counting ability makes the bridge between the initial capacity to perceive the numerosity and the more advanced arithmetic achievements. The learning of counting starts around the age of 2 years and continues until the age of 4 – 6 years, the children being able to count in an "adult" manner around the age of 6 years (Butterworth, 2005). Initially they learn the number – words and later they understand the "one – to – one correspondence", meaning that each of the number-words must be linked with only one object, no word must be used more than once and all objects must be counted. They also have to understand the "stable order principle" assuming that number-words must have a stably order sequence, the "cardinal principle" meaning that the number of objects is the last counting word used, the "abstraction principle" signifying that any set of items can be collected together for counting and the "order-irrelevance principle" assuming that they can start counting with any object in the set (Gelman & Gallistel, 1978).

Counting constitute the basis of the next steps in arithmetic cognition, namely addition and subtraction. Around the age of 7 – 8 years these operations are based on memory, counting strategies and other reconstructive strategies (Aschraft, 1982; Baroody, 1992). The multiplication tables are learned by drill at the age of 8 – 9 years, but with practice simple additions are also stored in memory. Later the children learn the other operations and the development of abstract thinking contribute to the understanding of arithmetic concepts and principles.

The arithmetic abilities are influenced by many factors such as culture, education and the lexicon (Dowker, 2005). The genetic factors also play a role in learning arithmetic, the mathematical disabilities can be present in genetic disorders like Turner's syndrome and Williams's syndrome. Furthermore, it was demonstrated that arithmetic disabilities have a greater concordance in monozygotic twins than in dizygotic twins (Knopnick et al., 1997).

Regarding the gender differences in arithmetic skills, although there is an opinion that males are better at mathematics than females, this statement must be regarded with certain reserve. There is a higher possibility that males are extremely good at mathematics, but at the other end of the scale, serious arithmetic difficulties are equally common in males and females (Dowker, 2005). Furthermore, there are studies that found no gender differences in arithmetic performances in kindergarten, first grade and even fifth-grade children in USA, Taiwan and Japan (Lummis & Stevenson, 1990).

In order to a better understanding of the development of arithmetic abilities, many researchers studied the acquisition disturbances of number processing and arithmetic; these disturbances are termed in the literature as dyscalculia, mathematics disorder (accordingly to Diagnostic and Statistical Manual of Mental Disorders - DSM IV) or arithmetical disorder (accordingly to The International Statistical Classification of Diseases and Related Health Problems – ICD-10). For example, Geary suggested as a basis for dyscalculia a deficit in working memory; in his view, poor working memory affects not only the learning of arithmetic facts (e.g. multiplication tables) but also the execution of calculation procedures (Geary, 1993). However, further studies demonstrated that although the working memory disturbances can coexist with calculation difficulties, they cannot be considered a causal factor as there is no statistical correlation between working memory and mathematic abilities (Temple & Sherwood, 2002). Geary and Hoard proposed that dyscalculia could be due to a semantic memory deficit (Geary & Hoard, 2001), but this hypothesis is contradicted by the studies showing that the semantic memory for numerical information and the semantic memory for non-numeric information are two different systems, with distinct cortical localization (Thioux et al., 1999).

The high co-morbidity between dyscalculia, aphasia-disphasia and/or dyslexia determined some researchers to investigate a possible link between the calculation disturbances and language disorders, but there were no qualitative (Shalev et al., 1997) or quantitative (Landerl et al., 2004) differences in number processing tests of children with these disabilities and children with dyscalculia alone.

All these studies sustained the conclusion that, although dyscalculia is frequently co-morbid with other cognitive disabilities, there is not a causal relationship between the disorders, dyscalculia being a specific problem with understanding basic numerical concepts, especially the numerosity concept (Butterworth, 2005).

The componential nature of arithmetic, ranging from counting to the ability to solve complex mathematic problems could explain the fact that almost any of these components can be affected relatively independently from the others. The studies in dyscalculia domain were not successful in establishing a strict hierarchy in the disturbances of these components and to find out if the disruption of any one component invariably precedes another. In the general population every aspect of arithmetic varies from severe deficits to high level of talent, some individuals showing strong discrepancies in either direction between almost any two components. Thereby, it is inadequate to label someone as globally "good" or "bad" at mathematic (Dowker, 2005).

Regarding the anatomic substrate of arithmetical processing, the neuroimaging studies show a greater activation of the prefrontal cortex (including the anterior cingulate cortex and the dorsolateral and ventrolateral regions) in children than in adults. In older subjects, there was observed an increased activation of the left parietal cortex, along the left supramarginal gyrus and the anterior intraparietal sulcus; also there was increased activity in the left lateral occipital and left temporal cortex. These data suggest that children necessitate more attention and working memory in order to achieve performances equal to adults. With increasing age, **the inferior parietal cortex specializes in mental arithmetic**, a process accompanied by a decreased dependency of calculation on memory and attention processes (Rivera et al., 2005; Ansari & Dhital, 2006).

Other areas that were activated during children's arithmetic calculation were basal ganglia, left hypocampus and parahypocampus, structures that are involved in memory processes. Furthermore, the basal ganglia could play a role in the motivational aspects (rewards and punishment mechanisms) and skills training (Haber, 2003).

The latest development of the neuroimaging techniques allowed the assumption of an ontogenetic and phylogenetic basis for number processing and a foundation on which are acquired the calculation abilities. Thus, Cantlon et al., demonstrated by functional neuroimaging (f-MRI) that the intraparietal sulcus is activated by numerical non-symbolic processing in both, children and adults and assumed that the intraparietal sulcus constitute the neural basis for number processing that preexists from childhood (Cantlon et al., 2006).

So far, numerical processing and calculation were investigated by scientists from various fields such as psychology, neurosciences and education. The cooperation between researchers in different disciplines will lead the integration of different results in a whole, fact that will permit a better understanding of neuropsychological and anatomical basis of arithmetical knowledge, with expansion of the diagnostic and rehabilitation opportunities in dyscalculia. Thus, the neuropsychological intervention based on knowledge of the many components of mathematical thinking will determine important progress. Furthermore, a better understanding of all the details in the development of the early arithmetic competencies and the later mathematic abilities will have a major impact in education.

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